

than 11.

28. Using distance formula prove that the points are collinear: A (4, -3, -1), B (5, -7, 6) and C (3, 1, -8). [3]

OR

Find the length of the medians of the triangle with vertices A(0, 0, 6), B (0, 4, 0) and C (6, 0, 0).

29. If the coefficients of a^{r-1} , a^r and a^{r+1} in the expansion of $(1 + a)^n$ are in arithmetic progression, prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$. [3]

OR

Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of

$$\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n \text{ is } \sqrt{6} : 1.$$

30. Express $(1 - 2i)^{-3}$ in the form of $(a + ib)$. [3]

OR

Find the multiplicative inverse of the complex number $= \sqrt{5} + 3i$

31. In a group of 500 persons, 300 take tea, 150 take coffee, 250 take cold drink, 90 take tea and coffee, 110 take tea and cold drink, 80 take coffee and cold drink and 50 take all the three drinks. [3]
- Find the number of persons who take none of the three drinks.
 - Find the number of persons who take only tea.
 - Find the number of persons who take coffee and cold drink but not tea.

Section D

32. A die is thrown. Describe the following events: [5]

- A: a number less than 7.
- B: a number greater than 7.
- C: a multiple of 3.
- D: a number less than 4.
- E: an even number greater than 4.
- F: a number not less than 3.

Also, find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, F' and $E \cap F'$.

33. Solve: $\lim_{x \rightarrow 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1}$ [5]

OR

Find the derivative of $x \sin x$ from first principle.

34. Find the sum of the following series up to n terms: [5]

- $5 + 55 + 555 + \dots$
- $6 + .66 + .666 + \dots$

35. If $A + B + C = \pi$, then prove that $\frac{\cos A}{\sin B \cdot \sin C} + \frac{\cos B}{\sin C \cdot \sin A} + \frac{\cos C}{\sin A \cdot \sin B} = 2$. [5]

OR

Prove that: $\cos 40^\circ \cos 80^\circ \cos 160^\circ = -\frac{1}{8}$.

Section E

36. **Read the text carefully and answer the questions:** [4]

Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.



- (i) Path traced by Arun represents which type of curve. Find the length of major axis?
- (ii) Find the equation of the curve traced by Arun?
- (iii) Find the eccentricity of path traced by Arun?

OR

Find the length of latus rectum for the path traced by Arun.

37. **Read the text carefully and answer the questions:**

[4]

An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

Particulars	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	₹ 5253	₹ 5253
Variance of the distribution of wages	100	121



- (i) Which firm A or B shows greater variability in individual wages?
- (ii) Find the standard deviation of the distribution of wages for firm B.
- (iii) Find the coefficient of variation of the distribution of wages for firm A.

OR

Find the amount paid by firm A.

38. **Read the text carefully and answer the questions:**

[4]

The purpose of the student council is to give students an opportunity to develop leadership by organizing and carrying out school activities and service projects. Create an environment where every student can voice out their concern or need. Raju, Ravi Joseph, Sangeeta, Priya, Meena and Aman are members of student's council.

There is a photo session in a school these 7 students are to be seated in a row for photo session.



- (i) Find the total number of arrangements so that Raju and Ravi are at extreme positions?
- (ii) Find the number of arrangements so that Joseph is sitting in the middle.

Solution

Section A

1.

(d) $\sin 17x - \sin 11x$

Explanation: We have, $2(1 - 2\sin^2 7x) \sin 3x = 2(\cos 14x) \sin 3x$

$[\because \cos 2x = 1 - 2\sin^2 x]$

$= 2 \cos 14x \cdot \sin 3x$

$= \sin 17x - \sin 11x$ [$\because 2 \cos A \sin B = \sin(A + B) - \sin(A - B)$]

$\therefore 2(1 - 2\sin^2 7x) \sin 3x = \sin 17x - \sin 11x$

2. (a) $R - \{-1/2, 1\}$

Explanation: Let $y = \frac{x^2 - x}{x^2 + 2x}$

$y(x^2 + 2x) = x^2 - x$

$yx(x+2) = x(x-1)$

$y(x+2) = x-1$

$x(y-1) = -(1+2y)$

$x = -\frac{(1+2y)}{y-1}$

Value of x can't be zero or it cannot be not defined. Therefore,

$y = 1, -1/2$ are not possible.

So, range = $R - \{-1/2, 1\}$

3.

(d) $\frac{1}{4}$

Explanation: $P(A) = \frac{1}{3}P(B)$

$\Rightarrow P(B) = 3P(A) \dots (1)$

A and B are mutually exclusive events.

$\Rightarrow P(A \cap B) = 0$

Now,

$P(A \cup B) = P(A) + P(B) = P(S)$

$\Rightarrow P(A) + P(B) = 1$

$\Rightarrow P(A) + 3P(A) = 1$ [From (1)]

$\Rightarrow 4P(A) = 1$

$\Rightarrow P(A) = \frac{1}{4}$

4. (a) $\cos 9$

Explanation: Given, $y = \frac{\sin(x+9)}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x \cdot \cos(x+9) - \sin(x+9)(-\sin x)}{\cos^2 x}$

$= \frac{\cos x \cos(x+9) + \sin x \sin(x+9)}{\cos^2 x}$

$= \frac{\cos(x+9-x)}{\cos^2 x} = \frac{\cos 9}{\cos^2 x}$

$\therefore \left(\frac{dy}{dx}\right)_{at x=0} = \frac{\cos 9}{\cos^2 0} = \frac{\cos 9}{(1)^2} = \cos 9$

5.

(b) $a = \pm\sqrt{2}b$

Explanation: Let D be the midpoint of BC. Hence its midpoint is $(\frac{0+a}{2}, 0) = (\frac{a}{2}, 0)$

Let E be the midpoint of AC. Hence its midpoint is $(\frac{0+a}{2}, \frac{0+b}{2}) = (\frac{a}{2}, \frac{b}{2})$

Slope of the median AD is $\frac{b-0}{(a/2)-0} = -\frac{2b}{a}$

Slope of BE = $\frac{(a/2)-0}{(b/2)-0} = \frac{b}{a}$

If BE and AD are perpendicular to each other then their product is -1



i.e; $-\frac{2b}{a} \times \frac{b}{a} = -1$

This implies $2b^2 = a^2$

$a = \pm\sqrt{2}b$

6.

(b) ϕ

Explanation: We have, $A \cap (A \cup B)' = A \cap (A' \cap B')$

$= (A \cap A') \cap (A \cap B')$

$= \phi \cap (A \cap B')$

$= \phi$

7. (a) $\left| \frac{\bar{z}}{z} \right|$

Explanation: $\left| \frac{\bar{z}}{z} \right|$

$\left| \frac{|\bar{z}|^2}{z\bar{z}} \right| = \left| \frac{|\bar{z}|^2}{|z|^2} \right| \left(\because z\bar{z} = |z|^2 \right)$

Let $z = a - ib$

$\Rightarrow |z| = \sqrt{a^2 + b^2}$

Let $\bar{z} = a + ib$

$\Rightarrow |\bar{z}| = \sqrt{a^2 + b^2}$

$\therefore \left| \frac{|\bar{z}|^2}{z\bar{z}} \right| = \left| \frac{|\bar{z}|^2}{|z|^2} \right|$

$= \left| \frac{\bar{z}}{z} \right|$

8.

(c) 2^{mn}

Explanation: Since we know that a relation from A to B consists of mn ordered pairs if they contain m and n elements respectively.

Each subset of those mn pairs will be a relation. so, each pair has two choices, either to be in that particular relation or not.

So, we have a total of 2^{mn} relations.

9.

(d) $x \in (10, \infty)$

Explanation: $-3x + 17 < -13$

$\Rightarrow -3x + 17 - 17 < -13 - 17$

$\Rightarrow -3x < -30$

$\Rightarrow \frac{-3x}{-3} > \frac{-30}{-3}$

$\Rightarrow x > 10$

$\Rightarrow x \in (10, \infty)$

10.

(d) 0

Explanation: $\cos 35^\circ + \cos 85^\circ + \cos 155^\circ$

$= 2 \cos \left(\frac{35^\circ + 85^\circ}{2} \right) \cos \left(\frac{35^\circ - 85^\circ}{2} \right) + \cos 155^\circ \left[\because \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$

$= 2 \cos 60^\circ \cos (-25^\circ) + \cos 155^\circ$

$= 2 \times \frac{1}{2} \cos 25^\circ + \cos 155^\circ$

$= \cos 25^\circ + \cos 155^\circ$

$= 2 \cos \left(\frac{25^\circ + 155^\circ}{2} \right) \cos \left(\frac{25^\circ - 155^\circ}{2} \right)$

$= 2 \cos 90^\circ \cos 65^\circ$

$= 0$

11. (a) N

Explanation: We have, $A' \cup (A \cup B) \cap B'$

$= A' \cup [(B' \cap A) \cup (B' \cap B)] \quad \{ \because \text{Distributive property of set: } (A \cap B) \cup (A \cap C) = A \cap (B \cup C) \}$

$= A' \cup [(A \cap B') \cup \Phi] \quad \{ \because (B' \cap B) = \phi \}$

$= A' \cup (A \cap B') = (A' \cup A) \cap (A' \cup B') \quad \{ \because \text{Distributive property of set: } (A \cup B) \cap (A \cup C) = A \cup (B \cap C) \}$

$$\begin{aligned}
&= \Phi \cap (A' \cup B') \{ \because (A' \cap A) = \phi \} \\
&= (A' \cup B') = (A \cap B)' \quad \{ \because (A' \cup B') \\
&= (A \cap B)' \} A' \cup (A \cup B) \cap B' = (A \cap B)'
\end{aligned}$$

A contains all odd numbers and B contains all even numbers

Therefore, $A \cap B = \phi$

$$\Rightarrow A' \cup (A \cup B) \cap B' = \{\phi\}'$$

$$\Rightarrow A' \cup (A \cup B) \cap B' = N$$

12.

(d) $\frac{1}{2}$

Explanation: Let the given geometric series be $(a + ar + ar^2 + \dots \infty)$

Then $ar = 2$ and $\frac{a}{(1-r)} = 8$

Putting, $r = \frac{2}{a}$, we get

$$\frac{a}{(1-\frac{2}{a})} = 8 \Rightarrow \frac{a^2}{(a-2)} = 8 \Rightarrow a^2 = 8a - 16$$

$$a^2 - 8a + 16 = 0 \Rightarrow (a - 4)^2 = 0 \Rightarrow a - 4 = 0 \Rightarrow a = 4.$$

$$\therefore r = \frac{2}{4} = \frac{1}{2}$$

Therefore, the required common ratio is $\frac{1}{2}$.

13.

(d) 5^n

Explanation: $\sum_{r=0}^n 4^r \cdot {}^n C_r = 4^0 \cdot {}^n C_0 + 4^1 \cdot {}^n C_1 + 4^2 \cdot {}^n C_2 + \dots + 4^n \cdot {}^n C_n$

$$= 1 + 4 \cdot {}^n C_1 + 4^2 \cdot {}^n C_2 + \dots + 4^n \cdot {}^n C_n$$

$$= (1 + 4)^n = 5^n$$

14.

(c) no solution

Explanation: $\frac{x+7}{x-8} > 2$

$$\Rightarrow \frac{x+7}{x-8} - 2 > 0$$

$$\Rightarrow \frac{x+7-2(x-8)}{x-8} > 0$$

$$\Rightarrow \frac{x+7-2x+16}{x-8} > 0$$

$$\Rightarrow \frac{(23-x)}{x-8} > 0 \quad [\because \frac{a}{b} > 0 \Rightarrow (a > 0 \text{ and } b > 0) \text{ or } (a < 0 \text{ and } b < 0)]$$

$$\Rightarrow (23 - x > 0 \text{ and } x - 8 > 0) \text{ or } (23 - x < 0 \text{ and } x - 8 < 0)$$

$$\Rightarrow (x < 23 \text{ and } x > 8) \text{ or } (x > 23 \text{ and } x < 8)$$

$$\Rightarrow 8 < x < 23 \quad [\text{Since } x > 23 \text{ and } x < 8 \text{ is not possible}]$$

$$\Rightarrow x \in (8, 23)$$

Now $\frac{2x+1}{7x-1} > 5$

$$\Rightarrow \frac{2x+1}{7x-1} - 5 > 0$$

$$\Rightarrow \frac{2x+1-5(7x-1)}{7x-1} > 0$$

$$\Rightarrow \frac{2x+1-35x+5}{7x-1} > 0$$

$$\Rightarrow \frac{(6-33x)}{7x-1} > 0 \quad [\because \frac{a}{b} > 0 \Rightarrow (a > 0 \text{ and } b > 0) \text{ or } (a < 0 \text{ and } b < 0)]$$

$$\Rightarrow (6 - 33x > 0 \text{ and } 7x - 1 > 0) \text{ or } (6 - 33x < 0 \text{ and } 7x - 1 < 0)$$

$$\Rightarrow (x < \frac{6}{33} \text{ and } x > \frac{1}{7}) \text{ or } (x > \frac{2}{11} \text{ and } x < \frac{1}{7})$$

$$\Rightarrow \frac{1}{7} < x < \frac{2}{11}$$

$$\Rightarrow x \in \left(\frac{1}{7}, \frac{2}{11} \right) \quad [\text{Since } x > \frac{2}{11} \text{ and } x < \frac{1}{7} \text{ is not possible}]$$

Hence, the solution of the system $\frac{x+7}{x-8} > 2, \frac{2x+1}{7x-1} > 5$ will be $(8, 23) \cap \left(\frac{1}{7}, \frac{2}{11} \right) = \phi$

15.

(d) $B^c \subset A^c$

Explanation: Let $A \subset B$

To prove $B^c \subset A^c$, it is enough to show that $x \in B^c \Rightarrow x \in A^c$

Let $x \in B^c$

$$\Rightarrow x \notin B$$

$$\Rightarrow x \notin A \text{ since } A \subset B$$

$$\Rightarrow x \in A^c$$

$$\text{Hence } B^c \subset A^c$$

16. (a) $\sin 4\beta$

Explanation: It is given that $\tan \alpha = \frac{1}{7}$ and $\tan \beta = \frac{1}{3}$

$$\text{Now, } \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta}$$

$$= \frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}}$$

$$= \frac{\frac{2}{3}}{\frac{8}{9}}$$

$$= \frac{3}{4}$$

$$\therefore \tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta}$$

$$= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = \frac{\frac{25}{28}}{\frac{25}{28}}$$

$$= 1$$

$$\tan(\alpha + 2\beta) = 1 = \tan \frac{\pi}{4}$$

$$\Rightarrow \alpha + 2\beta = \frac{\pi}{4}$$

$$\Rightarrow \alpha = \frac{\pi}{4} - 2\beta$$

$$\Rightarrow 2\alpha = \frac{\pi}{2} - 4\beta$$

$$\Rightarrow \cos 2\alpha = \cos\left(\frac{\pi}{2} - 4\beta\right) = \sin 4\beta$$

$$\therefore \cos 2\alpha = \sin 4\beta$$

17.

(c) z lies on the real axis

Explanation: $|w| = 1 \Rightarrow \left| \frac{1-iz}{z-i} \right| = 1$

$$\Rightarrow |1-iz| = |z-i|$$

$$\Rightarrow |1-i(x+iy)| = |(x+iy)-i|$$

$$\Rightarrow |(1+y)-ix| = |x+i(y-1)|$$

$$\Rightarrow \sqrt{(1+y)^2 + (x)^2} = \sqrt{(y-1)^2 + (x)^2}$$

$$\Rightarrow (1+y)^2 + (x)^2 = (y-1)^2 + (x)^2$$

$$\Rightarrow 1 + y^2 + 2y + x^2 = 1 + y^2 - 2y + x^2$$

$$\Rightarrow y = 0$$

Which is the real axis.

18.

(c) 60

Explanation: When E and I have 3 letters in between, which are possible in 1 way whereas other 3 letters are arranged in 3!,

$$\text{So, the number of arrangements} = 1 \times 3! = 6$$

$$\text{Thus, total number of arrangements} = 24 + 18 + 12 + 6 = 60$$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$$(1+x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 + \dots + n_{c_n}x^n$$

Reason:

$$(1+(-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + \dots + n_{c_n}(1)^{n-n}(-1)^n$$

$$= n_{c_0} - n_{c_1} + n_{c_2} - n_{c_3} + \dots + (-1)^n n_{c_n}$$

Each term will cancel each other

$$\therefore (1+(-1))^n = 0$$

Reason is also the but not the correct explanation of Assertion.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: True

Range = maximum value - minimum value

$R = L - S$

Reason: True

But not the correct explanation of Assertion.

Section B

21. i. $R = \{(4, 2) (6, 2) (8, 2) (6, 3) (9, 3) (8, 4)\}$

ii. Domain of $R = \{4, 6, 8, 9\}$

iii. Range of $R = \{2, 3, 4\}$

iv. $R^{-1} = \frac{1}{R} = \{(2, 4) (2, 6) (2, 8) (3, 6) (3, 9) (4, 8)\}$

OR

$$\text{Given, } |x - 2| = \begin{cases} x - 2, & x \geq 2 \\ -(x - 2), & x < 2 \end{cases} \text{ and } |x + 2| = \begin{cases} (x + 2), & x \geq -2 \\ -(x + 2), & x < -2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} (x - 2) + (2 + x), & 2 \leq x \leq 3 \\ -(x - 2) + (x + 2), & -2 \leq x < 2 \\ -(x - 2) - (x + 2), & -3 \leq x < -2 \end{cases}$$

$$= \begin{cases} x - 2 + 2 + x, & 2 \leq x \leq 3 \\ -x + 2 + x + 2, & -2 \leq x < 2 \\ -x + 2 - x - 2, & -3 \leq x < -2 \end{cases}$$

$$= \begin{cases} 2x, & 2 \leq x \leq 3 \\ 4, & -2 \leq x < 2 \\ -2x, & -3 \leq x < -2 \end{cases}$$

22. We have,

(LHL of $f(x)$ at $x = 1$)

$$= \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} 5(1 - h) - 4 = \lim_{h \rightarrow 0} 1 - 5h = 1$$

(RHL of $f(x)$ at $x = 1$)

$$= \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{h \rightarrow 0} f(1 + h) = \lim_{h \rightarrow 0} 4(1 + h)^3 - 3(1 + h) = 4(1)^3 - 3(1) = 1$$

$$\text{Clearly, } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x).$$

Hence, $\lim_{x \rightarrow 1} f(x)$ exists and is equal to 1.

23. The standard form of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Since the points (4, 3) and (-1, 4) lie on the ellipse, we have

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \dots \text{equation(1)}$$

$$\text{and } \frac{1}{a^2} + \frac{16}{b^2} = 1 \dots \text{equation(2)}$$

Solving equations (1) and (2), we find that $a^2 = \frac{247}{7}$ and $b^2 = \frac{247}{15}$.

Hence the required equation is $\frac{x^2}{\left(\frac{247}{7}\right)} + \frac{y^2}{\left(\frac{247}{15}\right)} = 1$, i.e., $7x^2 + 15y^2 = 247$.

OR

Given that, $5x^2 + 4y^2 = 1$

$$\Rightarrow \frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1$$

Which is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

where $a^2 = \frac{1}{5}$ and $b^2 = \frac{1}{4}$, i.e. $a = \frac{1}{\sqrt{5}}$ and $b = \frac{1}{2}$

Clearly $b > a$

$$\text{Now, } e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$\Rightarrow e = \sqrt{1 - \frac{\frac{1}{5}}{\frac{1}{4}}}$$

$$\Rightarrow e = \sqrt{1 - \frac{4}{5}}$$

$$\Rightarrow e = \frac{1}{\sqrt{5}}$$

$$\text{Coordinates of the foci} = (0, \pm be) = \left(0, \pm \frac{1}{2\sqrt{5}}\right)$$

$$\text{Length of the latus rectum} = \frac{2a^2}{b}$$

$$= \frac{2 \times \frac{1}{5}}{\frac{1}{2}}$$

$$= \frac{4}{5}$$

24. We know that, Natural numbers = 1, 2, 3, 4, 5, 6, ...

$$\text{If } x = 1, \text{ then } 2x + 3 = 2(1) + 3 = 2 + 3 = 5 \neq 4$$

\therefore no elements in the set B because the given equation $2x + 3 = 4$ is not satisfied for any natural number of x.

Hence, It is a null set.

25. The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-3 - 3}{-5 - 5} = \frac{-6}{-10}$$

$$m = \frac{3}{5}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y - 3 = \frac{3}{5}(x - 5) \Rightarrow 5(y - 3) = 3(x - 5)$$

$$3x - 15 - 5y + 15 = 0$$

$$3x - 5y = 0$$

Therefore, the required equation of line is $3x - 5y = 0$.

Section C

26. Here $f(x) = x^2$

$$\text{At } x = 1.1$$

$$f(1.1) = (1.1)^2 = 1.21$$

$$f(1) = (1)^2 = 1$$

$$\therefore \frac{f(1.1) - f(1)}{(1.1) - 1} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

27. Let x and x + 2 be two consecutive odd positive integers

$$\text{Then } x + 2 < 10 \text{ and } x + x + 2 > 11.$$

$$\Rightarrow x < 8 \text{ and } 2x + 2 > 11$$

$$\Rightarrow x < 8 \text{ and } 2x > 9$$

$$\Rightarrow x < 8 \text{ and } 2x > 9$$

$$\Rightarrow x < 8 \text{ and } x > \frac{9}{2}$$

$$\Rightarrow \frac{9}{2} < x < 8$$

$$\Rightarrow x = 5 \text{ and } 7$$

Thus required pairs of odd positive integers are 5, and 7.

$$28. AB = \sqrt{(5 - 4)^2 + (-7 + 3)^2 + (6 + 1)^2}$$

$$= \sqrt{(1)^2 + (-4)^2 + (7)^2}$$

$$= \sqrt{1 + 16 + 49}$$

$$= \sqrt{66}$$

$$BC = \sqrt{(3 - 5)^2 + (1 + 7)^2 + (-8 - 6)^2}$$

$$= \sqrt{(-2)^2 + (8)^2 + (-14)^2}$$

$$= \sqrt{4 + 64 + 196}$$

$$= \sqrt{264}$$

$$= 2\sqrt{66}$$

$$AC = \sqrt{(3 - 4)^2 + (1 + 3)^2 + (-8 + 1)^2}$$

$$= \sqrt{(-1)^2 + (4)^2 + (-7)^2}$$

$$= \sqrt{1 + 16 + 49}$$

$$= \sqrt{66}$$

$$\text{Here, } AB + AC = \sqrt{66} + \sqrt{66} = 2\sqrt{66} = BC$$

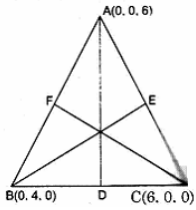
$$AB + AC = BC$$

Hence, the points are collinear.

OR

Here A(0, 0, 6), B(0, 4, 0) and C(6, 0, 0) are vertices of $\triangle ABC$

Now D is mid point of BC



$$\therefore \text{Coordinates of D is } \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right)$$

$$= (3, 2, 0)$$

$$\therefore AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2}$$

$$= \sqrt{9 + 4 + 36} = 7 \text{ units}$$

Also E is mid point of AC

$$\therefore \text{Coordinates of E is } \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{0+6}{2} \right)$$

$$= (3, 0, 3)$$

$$\therefore BE = \sqrt{(0-3)^2 + (4-0)^2 + (0-3)^2}$$

$$= \sqrt{9 + 16 + 9} = \sqrt{34} \text{ units}$$

Also F is mid point of AB

$$\therefore \text{Coordinates of F is } \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right)$$

$$= (0, 2, 3)$$

$$\therefore CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36 + 4 + 9} = 7 \text{ units}$$

29. We have the $(r+1)^{\text{th}}$ term in the expansion is ${}^n C_r a^r$. Therefore, it can be seen that a^r occurs in the $(r+1)^{\text{th}}$ term, and its coefficient is ${}^n C_r$. Therefore, the coefficients of a^{r-1} , a^r and a^{r+1} are ${}^n C_{r-1}$, ${}^n C_r$ and ${}^n C_{r+1}$, respectively. Since these coefficients are in arithmetic progression, so we have, ${}^n C_{r-1} + {}^n C_{r+1} = 2 \cdot {}^n C_r$ it is given

$$\frac{n!}{(r-1)!(n-r+1)!} + \frac{n!}{(r+1)!(n-r-1)!} = 2 \times \frac{n!}{r!(n-r)!}$$

$$\text{i.e. } \frac{1}{(r-1)!(n-r+1)(n-r)(n-r-1)!} + \frac{1}{(r+1)(r)(r-1)!(n-r-1)!} = 2 \times \frac{1}{r(r-1)!(n-r)(n-r-1)!}$$

$$\text{OR } \frac{1}{(r-1)!(n-r-1)!} \left[\frac{1}{(n-r)(n-r+1)} + \frac{1}{(r+1)(r)} \right] = 2 \times \frac{1}{(r-1)!(n-r-1)! [r(n-r)]}$$

$$\text{i.e. } \frac{1}{(n-r+1)(n-r)} + \frac{1}{r(r+1)} = \frac{2}{r(n-r)}$$

$$\text{OR } \frac{r(r+1) + (n-r)(n-r+1)}{(n-r)(n-r+1)r(r+1)} = \frac{2}{r(n-r)}$$

$$\text{OR } r(r+1) + (n-r)(n-r+1) = 2(r+1)(n-r+1)$$

$$\text{OR } r^2 + r + n^2 - nr + n - nr + r^2 - r = 2(nr - r^2 + r + n - r + 1)$$

$$\text{OR } n^2 - 4nr - n + 4r^2 - 2 = 0$$

$$\text{so, } n^2 - n(4r+1) + 4r^2 - 2 = 0$$

OR

$$\text{We have } \left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}} \right)^n$$

$$5^{\text{th}} \text{ term from the beginning} = {}^n C_4 (\sqrt[4]{2})^{n-4} \left(\frac{1}{\sqrt[4]{3}} \right)^4$$

$$5^{\text{th}} \text{ term from the end} = (n+1-5+1)^{\text{th}} \text{ term from beginning}$$

$$= (n-3)^{\text{th}} \text{ term from beginning}$$

$$= {}^n C_{n-4} (\sqrt[4]{2})^4 \left(\frac{1}{\sqrt[4]{3}} \right)^{n-4}$$

$$\text{Now } \frac{{}^nC_4(\sqrt[4]{2})^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4}{{}^nC_{n-4}(\sqrt[4]{2})^4\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}} = \frac{\sqrt{6}}{1}$$

$$\Rightarrow (2)^{\frac{n-8}{4}} \cdot (3)^{\frac{n-8}{4}} = 2^{\frac{1}{2}} \times 3^{\frac{1}{2}}$$

$$\frac{n-8}{4} = \frac{1}{2} \Rightarrow n - 8 = 2$$

$$\Rightarrow n = 10$$

30. Let $z = (1 - 2i)^{-3}$

$$= \frac{1}{(1-2i)^3} = \frac{1}{1-8i^3-6i+12i^2} \quad [\because (a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2]$$

$$= \frac{1}{1-8i^2-i-6i+12(-1)}$$

$$= \frac{1}{1+8i-6i-12} \quad [\because i^2 = -1]$$

$$= \frac{1}{-11+2i} = \frac{1}{-11+2i} \times \frac{-11-2i}{-11-2i} \quad [\text{multiplying numerator and denominator by } -11-2i]$$

$$= \frac{-11-2i}{(-11)^2-(2i)^2} = \frac{-11-2i}{121+4} \quad [\because (a-b)(a+b) = a^2 - b^2]$$

$$= \frac{-11-2i}{125} = \frac{-11}{125} - \frac{2i}{125} = a + ib \quad [\text{say}]$$

where, $a = \frac{-11}{125}$ and $b = \frac{-2}{125}$

OR

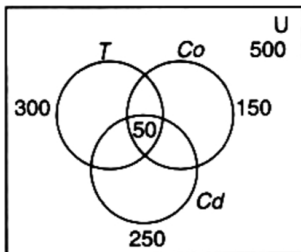
M.I. of $= \sqrt{5} + 3i$

$$= \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$

$$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$$

$$= \frac{\sqrt{5}-3i}{5-9i^2} = \frac{\sqrt{5}-3i}{5+9} = \frac{1}{14}(\sqrt{5}-3i)$$

31. Given,
- $n(U) = 500, n(T) = 300, n(Co) = 150, n(Cd) = 250,$
 $n(T \cap Co) = 90, n(T \cup Cd) = 110,$
 $n(Co \cap Cd) = 80, n(T \cap Cd \cap Co) = 50$



Here, $n(T)$ = like tea,
 $n(Co)$ = like coffee,
 $n(Cd)$ = like cold drink

- i. Number of persons who take none of three drinks

$$n(U) - n(T \cup Co \cup Cd)$$

$$n(T \cup Co \cup Cd) = n(T) + n(Co) + n(Cd) - n(T \cap Co)$$

$$- n(T \cap Cd) - n(Co \cap Cd) + n(T \cap Cd \cap Co)$$

$$= 300 + 150 + 250 - 90 - 110 - 80 + 50$$

$$= 750 - 280$$

$$= 470$$

$$n(U) - n(T \cup Co \cup Cd) = 500 - 470 = 30$$

Number of persons who take none of three drinks = 30.

- ii. Number of person who take only tea

$$= n(T) - n(T \cap Co) - n(T \cap Cd) + n(T \cap Cd \cap Co)$$

$$= 300 - 90 - 110 + 50$$

$$= 350 - 200 = 150$$

\therefore Number of person who take only tea = 150

- iii. Number of persons who take coffee and cold drink but not tea

$$= n(Co \cap Cd) - n(T \cap Co \cap Cd)$$

$$= 80 - 50 = 30$$

∴ Number of persons who take coffee and cold drink but not tea = 30

Section D

32. When a die is thrown, then sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

i. A: a number less than 7 = $\{1, 2, 3, 4, 5, 6\}$

ii. B: a number greater than 7 = $\{\} = \phi$

iii. C: a multiple of 9 = $\{3, 6\}$

iv. D: a number less than 4 = $\{1, 2, 3\}$

v. E: an even number greater than 4 = $\{6\}$

vi. F: a number not less than 3 = $\{3, 4, 5, 6\}$

Now, $A \cup B =$ The elements which are in A or B or both

$$= \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$$

$A \cap B =$ The elements which are common in both A = B

$$= \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$$

$B \cup C =$ The elements which are in B or C or both

$$= \{\} \cup \{3, 6\} = \{3, 6\}$$

$E \cap F =$ The elements which are common in both E and F

$$= \{6\} \cap \{3, 4, 5, 6\} = \{6\}$$

$D \cap E =$ The elements which are common in both D and E

$$= \{1, 2, 3\} \cap \{6\} = \phi$$

$A - C =$ The elements which are in A but not in C

$$= \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$$

$D - E =$ The elements which are in D but not in E

$$= \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$$

$$F' = (S - F) = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$$

and $E \cap F' = E \cap (S - F)$

$$= \{6\} \cap \{1, 2\} = \phi$$

33. Dividing $x^4 - 3x^3 + 2$ by $x^3 - 5x^2 + 3x + 1$

$$\begin{array}{r} x^3 - 5x^2 + 3x + 1 \overline{) x^4 - 3x^3 + 2} \\ \underline{\pm x^4 \mp 5x^3 \pm 3x^2 \pm x} \\ 2x^3 - 3x^2 - x + 2 \\ \underline{\pm 2x^3 \mp 10x^2 \pm 6x \pm 2} \\ 7x^2 - 7x \end{array}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1} = \lim_{x \rightarrow 1} (x + 2) + \lim_{x \rightarrow 1} \frac{7x^2 - 7x}{x^3 - 5x^2 + 3x + 1}$$

$$= \lim_{x \rightarrow 1} x + 2 + \lim_{x \rightarrow 1} \frac{7x(x-1)}{x^3 - 5x^2 + 3x + 1}$$

$$= \lim_{x \rightarrow 1} x + 2 + \lim_{x \rightarrow 1} \frac{7x(x-1)}{(x-1)(x^2 - 4x - 1)}$$

$$= \lim_{x \rightarrow 1} x + 2 + \lim_{x \rightarrow 1} \frac{7x}{(x^2 - 4x - 1)}$$

$$= 1 + 2 + \frac{7}{(1 - 4 - 1)}$$

$$= 3 - \frac{7}{4}$$

$$= \frac{12 - 7}{4}$$

$$= \frac{5}{4}$$

OR

We have, $f(x) = x \sin x$

By using first principle of derivative,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h) \sin(x+h) - x \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)[\sin x \cos h + \cos x \sin h] - x \sin x}{h} \quad [\because \sin(x+y) = \sin x \cos y + \cos x \sin y]$$

$$= \lim_{h \rightarrow 0} \frac{[x \sin x \cos h + x \cos x \sin h + h \sin x \cos h + h \cos x \sin h - x \sin x]}{h}$$



$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{[x \sin x (\cos h - 1) + x \cdot \cos x \cdot \sin h + h(\sin x \cdot \cos h + \cos x \cdot \sin h)]}{h} \\
&= \lim_{h \rightarrow 0} \frac{x \sin x (\cos h - 1)}{h} + \lim_{h \rightarrow 0} x \cdot \cos x \cdot \frac{\sin h}{h} + \lim_{h \rightarrow 0} \frac{h(\sin x \cdot \cos h + \cos x \cdot \sin h)}{h} \\
&= x \sin x \lim_{h \rightarrow 0} \left[\frac{-(1 - \cos h)}{h} \right] + x \cos x + \sin x \\
&= -2x \sin x \cdot \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin^2 \frac{h}{2}}{h \times \frac{h}{4}} \times \frac{h}{4} + x \cos x + \sin x \\
&= -x \cdot \sin x \cdot \frac{2}{4} \lim_{\frac{h}{2} \rightarrow 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \times h + x \cos x + \sin x \\
&= -x \sin x \cdot \frac{1}{2} (1) \times 0 + x \cos x + \sin x \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&= x \cos x + \sin x
\end{aligned}$$

34. i. $S_n = 5 + 55 + 555 + \dots$ up to n terms

$$\begin{aligned}
&= 5 [1 + 11 + 111 + \dots \text{ up to } n \text{ terms}] \\
&= \frac{5}{9} [9 + 99 + 999 + \dots \text{ up to } n \text{ terms}] \\
&= \frac{5}{9} [(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots \text{ up to } n \text{ terms}] \\
&= \frac{5}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right] \\
&= \frac{5}{9} \left[\frac{10}{9} (10^n - 1) - n \right] \\
&= \frac{50}{81} (10^n - 1) - \frac{5}{9} n
\end{aligned}$$

ii. $S_n = .6 + .66 + .666 + \dots$ up to n terms

$$\begin{aligned}
&= 6 [.1 + .11 + .111 + \dots \text{ up to } n \text{ terms}] \\
&= \frac{6}{9} [.9 + .99 + .999 + \dots \text{ up to } n \text{ terms}] \\
&= \frac{6}{9} \left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \text{ up to } n \text{ terms} \right] \\
&= \frac{6}{9} \left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \dots \text{ up to } n \text{ terms} \right] \\
&= \frac{6}{9} \left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots \text{ up to } n \text{ terms}\right) \right] \\
&= \frac{2}{3} \left[n - \frac{\frac{1}{10} \left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}} \right] \\
&= \frac{2}{3} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n}\right) \right] \\
&= \frac{2n}{3} - \frac{2}{27} \left(1 - \frac{1}{10^n}\right)
\end{aligned}$$

35. Given, $A + B + C = \pi$

$$\Rightarrow A = \pi - (B + C) \dots (i)$$

$$\text{Now, } \frac{\cos A}{\sin B \cdot \sin C} = \frac{\cos[\pi - (B + C)]}{\sin B \cdot \sin C}$$

$$= \frac{-\cos(B + C)}{\sin B \cdot \sin C} \quad [\because \cos(\pi - \theta) = -\cos \theta]$$

$$= \frac{-[\cos B \cdot \cos C - \sin B \cdot \sin C]}{\sin B \cdot \sin C}$$

$$= -[\cot B \cot C - 1]$$

$$\therefore \frac{\cos A}{\sin B \cdot \sin C} = 1 - \cot B \times \cot C \dots (ii)$$

$$\text{Similarly, } \frac{\cos B}{\sin C \cdot \sin A} = 1 - \cot A \times \cot C \dots (iii)$$

$$\text{and } \frac{\cos C}{\sin A \cdot \sin B} = 1 - \cot A \times \cot B \dots (iv)$$

On adding Eqs. (ii), (iii) and (iv), we get

$$\begin{aligned}
&\frac{\cos A}{\sin B \cdot \sin C} + \frac{\cos B}{\sin C \cdot \sin A} + \frac{\cos C}{\sin A \cdot \sin B} \\
&= 3 - (\cot B \times \cot C + \cot A \times \cot C + \cot A \times \cot B) \dots (v)
\end{aligned}$$

$$\text{But } \cot(A + B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$$

$$\Rightarrow \cot(\pi - C) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$$

$$[\because A + B + C = \pi]$$

$$[\because A + B = \pi - C]$$

$$\Rightarrow -\cot C = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A} \quad [\because \cot(\pi - \theta) = -\cot \theta]$$

$$\Rightarrow -\cot C [\cot B + \cot A] = \cot A \cot B - 1$$

$$\Rightarrow -(\cot B \cot C + \cot C \cot A) - \cot A \cot B = -1$$

$$\Rightarrow \cot B \times \cot C + \cot C \times \cot A + \cot A \times \cot B = 1 \dots(vi)$$

On putting (vi) value in Eq. (v), we get,

$$\frac{\cos A}{\sin B \cdot \sin C} + \frac{\cos B}{\sin C \cdot \sin A} + \frac{\cos C}{\sin A \cdot \sin B} = 3 - 1 = 2$$

Hence proved.

OR

$$\cos 40^\circ \cos 80^\circ \cos 160^\circ = -\frac{1}{8}$$

$$\text{LHS} = \cos 40^\circ \cos 80^\circ \cos 160^\circ$$

$$= \cos 80^\circ \cos 40^\circ \cos 160^\circ$$

Multiplying and dividing by 2

$$= \frac{1}{2} \{\cos 80^\circ \times (2 \cos 40^\circ \cos 160^\circ)\}$$

$$\text{Because } 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2} \cos 80^\circ [\cos(40^\circ + 160^\circ) + \cos(40^\circ - 160^\circ)]$$

$$= \frac{1}{2} \cos 80^\circ [\cos 200^\circ + \cos(-120^\circ)]$$

$$= \frac{1}{2} \cos 80^\circ [\cos 200^\circ + \cos 120^\circ]$$

$$= \frac{1}{2} \cos 80^\circ \{\cos(180^\circ + 20^\circ) + \cos(180^\circ - 60^\circ)\}$$

$$= \frac{1}{2} \cos 80^\circ (-\cos 20^\circ - \cos 60^\circ)$$

$$= -\frac{1}{2} \cos 80^\circ \cos 20^\circ - \frac{1}{2} \cos 80^\circ \cos 60^\circ$$

$$= -\frac{1}{4} (2 \cos 80^\circ \cos 20^\circ) - \frac{1}{4} \cos 80^\circ$$

$$= -\frac{1}{4} [2 \cos 80^\circ \cos 20^\circ + \cos 80^\circ]$$

$$= -\frac{1}{4} [\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ) + \cos 80^\circ]$$

$$= -\frac{1}{4} [\cos 100^\circ + \cos 60^\circ + \cos 80^\circ]$$

$$= -\frac{1}{4} [\cos(180^\circ - 80^\circ) + \cos 60^\circ + \cos 80^\circ]$$

$$= -\frac{1}{4} [-\cos 80^\circ + \cos 60^\circ + \cos 80^\circ]$$

$$= -\frac{1}{4} \cos 60^\circ$$

$$= -\frac{1}{4} \times \frac{1}{2}$$

$$= -\frac{1}{8} = \text{RHS}$$

Section E

36. Read the text carefully and answer the questions:

Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.



- (i) An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Hence path traced by Arun is ellipse.

Sum of the distances of the point moving point to the foci is equal to length of major axis = 10m

- (ii) Given $2a = 10$ & $2c = 8$

$$\Rightarrow a = 5 \text{ \& } c = 4$$

$$c^2 = a^2 + b^2$$

$$\Rightarrow 16 = 25 + b^2$$

$$\Rightarrow b^2 = 25 - 16 = 9$$

$$\text{Equation of ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Required equation is } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

- (iii) equation is of given curve is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a = 5$, $b = 3$ and given $2c = 8$ hence $c = 4$



$$\text{Eccentricity} = \frac{c}{a} = \frac{4}{5}$$

OR

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Hence $a = 5$ and $b = 3$

$$\text{Length of latus rectum of ellipse is given by } \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$$

37. Read the text carefully and answer the questions:

An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

Particulars	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	₹ 5253	₹ 5253
Variance of the distribution of wages	100	121



- (i) coefficient of variation of wages, of firm A = 0.19
 coefficient of variation of wages, of firm B = $\frac{121}{5253} \times 100 = 0.21$
 \therefore Firm B shows greater variability in individual wages.
- (ii) Standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{121} = 11$
- (iii) Variance of distribution of wages, $\sigma^2 = 100$
 Standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{100} = 10$
 coefficient of Variation = $\frac{\sigma}{\bar{x}} \times 100$
 $= \frac{10}{5,253} \times 100$
 $= 0.19$

OR

No. of wage earners = 586

Mean of monthly wages, $\bar{x} = ₹5253$

Amount paid by firm A = ₹(586 × 5253) = ₹3078258

38. Read the text carefully and answer the questions:

The purpose of the student council is to give students an opportunity to develop leadership by organizing and carrying out school activities and service projects. Create an environment where every student can voice out their concern or need. Raju, Ravi Joseph, Sangeeta, Priya, Meena and Aman are members of student's council. There is a photo session in a school these 7 students are to be seated in a row for photo session.



- (i) Given Raju and Ravi are at the extreme positions
Case 1 Raju _____ Ravi
Case 2 Ravi _____ Raju
 So remaining 5 places are filled in 5! Ways in both cases
 $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
 Hence total number of arrangements = $120 \times 2 = 240$ ways
- (ii) _____ **Joseph** _____
 So here middle place is occupied by Joseph remaining 6 places are filled by remaining 6 students in 6! Ways
 $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways